Sample Paper 2014: Paper 1

Question 5 (25 marks)

Question 5 (a)

Sketch the function by finding its turning points and end points.

$f(x) = x^3 - 5x^2 + 3x + 5$

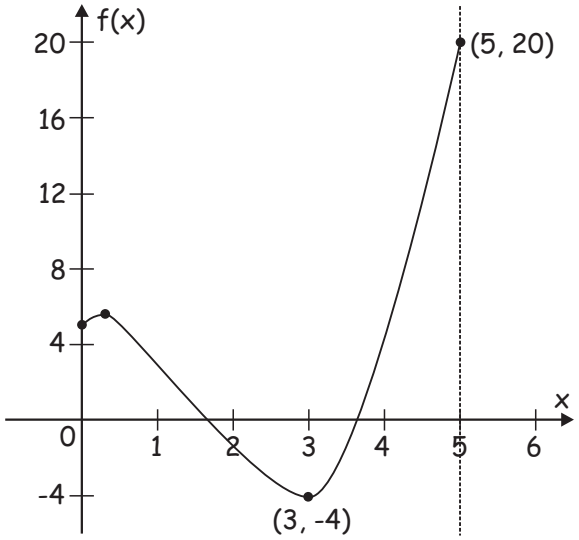
$f'(x) = 3x^2 - 10x + 3$

Turning points: $f'(x) = 0$

$3x^2 - 10x + 3 = 0$

$(3x - 1)(x - 3) = 0$

$x = \frac{1}{3},\ 3$



$f(\frac{1}{3}) = (\frac{1}{3})^3 - 5(\frac{1}{3})^2 + 3(\frac{1}{3}) + 5 = 0 \Rightarrow (\frac{1}{3},\ \frac{148}{27})$ is a turning point.

$f(3) = (3)^3 - 5(3)^2 + 3(3) + 5 = 0 \Rightarrow (3,\ -4)$ is a turning point.

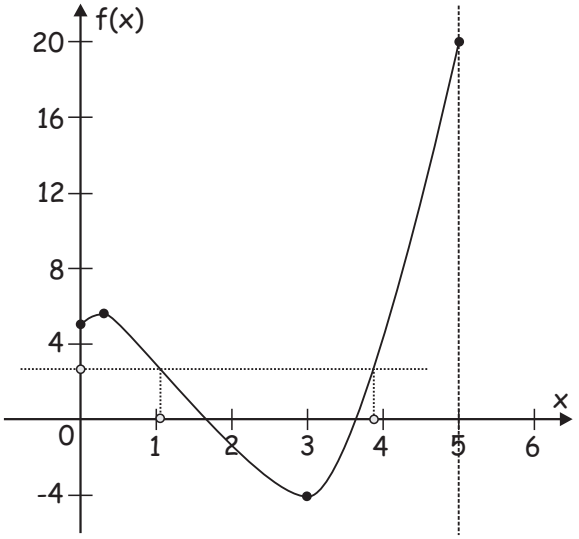
$x = 0 : f(0) = (0)^3 - 5(0)^2 + 3(0) + 5 = 5 \Rightarrow (0,\ 5)$ is the starting point.

$x = 5 : f(5) = (5)^3 - 5(5)^2 + 3(5) + 5 = 20 \Rightarrow (5,\ 20)$ is the finishing point.

Answer: Maximum value of $f = 20$
Minimum value of $f = -4$

Question 5 (b)

The function f is **not** injective.
An injective function never maps distinct elements of its domain to the same element of its range. Each element in the domain must map on to a unique element in the range.
Look at the sketch of a diagram of the function f.



You can see from the sketch that the function is not injective as two points are shown on the x-axis which map on to the same point on the $f(x)$ axis.